Topic: Intermediate value theorem with an interval

Question: Which of the following statements is true?

Answer choices:

A The Intermediate Value Theorem only applies to discontinuous functions.

B The Intermediate Value Theorem only applies when there is no interval.

C The Intermediate Value Theorem only applies when there is an open interval.

D The Intermediate Value Theorem only applies when there is a closed interval.

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Solution: D

The Intermediate Value Theorem states that for a function on a closed interval $[a, b]$ where the function is continuous on the interval, a point c exists on the interval where $f(c) = k$.

$f(a) < k < f(b)$ and $a < c < b$

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Answer choices:

A The Intermediate Value Theorem states that for a function on a closed interval $[a, b]$ where the function is continuous on the interval a point c exists on the interval where the function equals k.

B The Intermediate Value Theorem states that for a function on an open interval (a, b) where the function is continuous on the interval a point c exists on the interval where the function equals k.

C The Intermediate Value Theorem states that for a function on a closed interval $[a, b]$ where the function is discontinuous on the interval a point c exists on the interval where the function equals k.

D The Intermediate Value Theorem states that for a function on an open interval (a, b) where the function is discontinuous on the interval a point c exists on the interval where the function equals k.

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Solution: A

The Intermediate Value Theorem states that for a function on a closed interval $[a, b]$ where the function is continuous on the interval, a point c exists on the interval where $f(c) = k$.

$f(a) < k < f(b)$ and $a < c < b$

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Topic: Intermediate value theorem with an interval

Question: Is there a root for the function on the given interval?

$$f(x) = x^2 - 4$$

on the interval $[1,6]$

Answer choices:

A Yes, there's a root at $(0,4)$.

B Yes, there's a root at $(0, -4)$.

C Yes, there's a root at $(2,0)$.

D Yes, there's a root at $(-2,0)$.

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Solution: C

A root is a place where the function crosses the x-axis, or where $y = 0$. The given function

$$f(x) = x^2 - 4$$

is continuous on the interval $[1,6]$ (you can confirm this graphically if needed), and the interval is closed, so we can apply the IVT.

Next we can use the end-points of the interval to find the y-values (range).

$$f(1) = (1)^2 - 4$$

$$f(1) = 1 - 4$$

$$f(1) = -3$$

and

$$f(6) = (6)^2 - 4$$

$$f(6) = 36 - 4$$

$$f(6) = 32$$

This means the range of the function is $[-3,32]$.

The IVT confirms then that the function has a root, as the range includes 0. The root exists at $0 = x^2 - 4$ or $x = 2$.

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